VEF MATHEMATICS SUBJECT TEST

BACKGROUND

VEF Subject Tests are designed to assess mastery of concepts, principles, and knowledge expected of applicant at the conclusion of an academic major in specific subject areas. In addition to factual knowledge, the tests evaluate applicant's abilities to analyze and solve problems, understand relationships, and interpret material. There will also be a set of basic mathematical questions. The tests may contain questions that require interpretation of graphs, diagrams, and charts based on material related to the field.

These VEF Subject Tests are three-hour multiple-choice tests.

The Subject Tests score reports will be sent directly to VEF. Applicants will be notified by VEF Offices.

VEF MATHEMATICS SUBJECT TESTS

The Major Field Test in Mathematics consist of questions, some of which may be grouped in sets and based on such materials as diagrams and graphs. The questions are drawn from the courses of study most commonly offered as part of an undergraduate mathematics curriculum.

The outline below shows the content areas covered on the test of questions among the areas.

I. Calculus

(The usual material of 3 semesters of calculus including single and multivariable calculus.)

II. Algebra

A. Linear Algebra
1. Matrices
2. Linear transformations
3. Characteristic polynomials
4. Eigenvalues and eigenvectors
5. Vector spaces
6. Systems of linear equations

B. Abstract Algebra
1. Elementary theory of groups, rings, and fields
2. Elementary topics from number theory

III. Additional Topics

A. Advanced calculus (The standard subjects of an advanced calculus/beginning analysis course, including limits, Cauchy sequences, and general convergence of sequences, series and functions. All problems in either single or multivariable calculus are classified as regular calculus problems.)
B. Real analysis (including topology of the real line)
C. Discrete mathematics (graph theory and combinatorics)
D. Probability and statistics
E. Dynamical systems
F. Point-set topology
G. Geometry (Euclidean, non-Euclidean, and differential geometry)
H. Differential equations (The standard subjects in a one semester course on differential equations that are not covered in the calculus sequence. Problems such as exponential decay are classified as calculus problems.)
I. Numerical analysis J. Complex analysis

The questions at various cognitive levels is as follows:

i. Routine

Involves only two or three definitions and no more than a two-step reasoning process; or involves standard techniques normally taught and practiced extensively in a course that is generally required or strongly recommended for all math majors at most institutions.

ii. Nonroutine

Includes all items that are considered insightful. Also includes items that require several steps of reasoning and items that require either the use of several definitions or a "new" definition which the student would not be expected to know. Some questions may require bringing techniques from two or more areas to bear on one problem, *e.g.*, treating functions from calculus as elements of an algebraic system.

iii. Applied

There is conceptual overlap between "Applied" and "Routine-Nonroutine". Into which of these categories a given question is placed depends primarily on the general nature of the question. For example, all "real world" settings are placed in the "Applied" category. On the other hand, standard applications of one area of mathematics to another (such as using differential calculus to solve geometric problems about the slope of tangent lines to curves) would not be placed in the "Applied" category

SAMPLE QUESTIONS FOR VEF SUBJECT TEST IN MATHEMATICS

The following questions illustrate the range of the test in terms of the abilities measured, the disciplines covered, and the difficulty of the questions posed. They should not, however, be considered representative of the entire scope of the test in either content or difficulty. An answer key follows the questions.

1. On a questionnaire, a respondent must choose 3 of the 5 questions presented. How many different combinations of 3 questions can the respondent possibly choose?

(A) 10
(B) 15
(C) 20
(D) 30
(E) 60

2. A function *f* has the property that, at every point (x, y) on the curve $y = f(x)$, the slope of the line tangent to the curve is equal to $2xy$. Which of the following best describes the function *f* ?

(A) Linear
(B) Trigonometric
(C) Inverse trigonometric
(D) Logarithmic
(E) Exponential

3. Let *A* and *B* be metric spaces and let $f: A \to B$. Suppose that whenever *X* is an open set in *B*, the set $\{ a \in A : f(a) \notin X \}$ is closed in *A*. Which of the following must be true?

I. *f* is injective
II. *f* is continuous.
III. *f* is a homeomorphism.

(A) None
(B) II only
(C) III only
(D) I and III only
(E) I, II, and III

4. In the *xy*-plane, the line that is tangent to the graph of $y = x^2$ at the point (2, 4) has a slope of

(A) 1/2
(B) 1
(C) 2
(D) 4
(E) 8

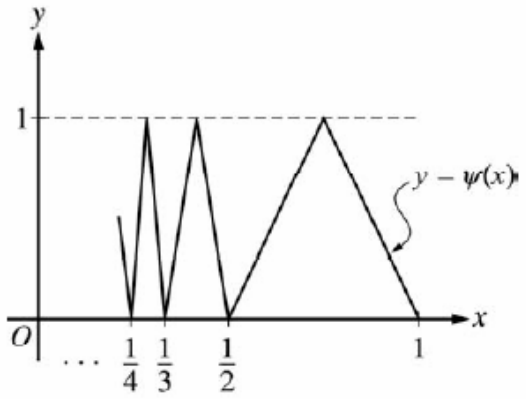
5. The set $\{1, 2, 4, 7, 8, 11, 13, 14\}$ forms a group under the operation of multiplication modulo 15. Which of the following is the cyclic subgroup generated by $\{7\}$?

(A) $\{1, 7\}$
(B) $\{1, 2, 7, 14\}$
(C) $\{1, 4, 7, 13\}$
(D) $\{1, 7, 8, 13, 14\}$
(E) $\{4, 7, 11, 14\}$

6. For each real number $t \neq 0$, define the function ϕt: $R \to R$ by $\phi_t(x) = |x|^{|t|}$. A subset A of real numbers is called invariant with respect to the collection of functions ϕ_t if $\phi_t(A) \subseteq A$ for each $t \neq 0$. For this collection of functions, which of the following intervals are invariant?

I. (0,1]
II. [0, 1/2)
III.(0, ∞)

(A) I only
(B) II only
(C) III only
(D) I and III
(E) II and III



7**. A portion of the graph of a continuous nonnegative function $y = \psi(x)$ is shown above, where $\psi(0) = 0$ and $\psi(1/n) = 0$ for each positive integer n. If the graph of $y = \psi(x)$ between $x = 1/(n+1)$ and $x = 1/n$ consists of the congruent sides of an isosceles triangle of height 1 for each positive integer n, then $\int_0^1 \psi(x)dx =$**

(A) $\frac{1}{2}$
(B) $\frac{1}{4}$
(C) $\frac{1}{\pi}$
(D) $\frac{1}{e}$
(E) $\frac{2}{e}$

8. The function f is differentiable on the interval $0 < x < 4$. If $f(1) = 1$ and $f(3) = 7$, then for some $1 < c < 3$, $f'(c)$ must be equal to

(A) 1
(B) 2
(C) 3
(D) 4
(E) 6

9. If a and b are integers, how many matrices of the form $\begin{bmatrix} 2 & a \\ b & 3 \end{bmatrix}$ are <u>not</u> invertible?

(A) One
(B) Two
(C) Four
(D) Eight
(E) More than eight

10. If V_n is the real vector space of all n-tuples of real numbers for each $n > 1$, which of the following must be true?

I. Every basis of V_n contains exactly n vectors.
II. Every basis of V_n is an orthogonal set of vectors.
III. Every set of $n + 1$ vectors of V_n is a linearly dependent set.

(A) I only
(B) II only
(C) I and II
(D) I and III

(E) II and III

ANSWER KEY

1. A
2. E
3. B
4. D
5. C
6. D
7. A
8. C
9. D
10. D
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